Sampling waveforms and computing spectra

by Don Southwick
Technical Training Engineer
Bently Nevada Corporation

There are three "Laws of the Universe" which deal with sampling waveforms and computing spectra with a Fast Fourier Transform (FFT):

- The sample rate governs the frequency response.
- The total sample time governs the resolution.
- The number of samples governs the number of lines.

Since sample rate, total sample time and the number of samples interact, you need to only specify two of the three.

Law #1: The sample rate governs the frequency response

The faster you sample, the higher the frequency response you will get. Or, conversely, if you need to measure high frequencies, the sample rate must also be high. In fact, to avoid a phenomenon known as aliasing, the sample rate must be at least twice as fast as the highest frequency component in the signal.

$$F_S \geq 2 \cdot F_{Max}$$

$$F_S = 1/t_S$$

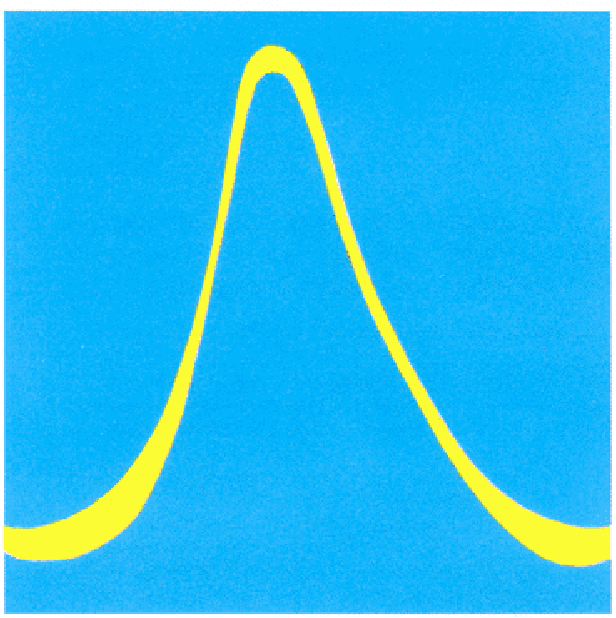
where F_S = the sample rate in Hertz

F_{Max} = the highest frequency component in the signal in Hertz

t_S = the sample interval in seconds

That seems easy enough, but how do you know the highest frequency component in a signal you haven't measured yet? The answer is that you don't know. There is, however, a way around this quandary: first, specify the highest frequency you want to measure, F_{Max}. Then, remove all frequency components in the signal higher than F_{Max} with a low-pass filter (which is called an anti-alias filter in this application). Finally, the low-pass filtered signal can be sampled at a rate faster than $2 \cdot F_{Max}$ (2.5 to 3 times is typical).

Since "real-world" filters must be used (theoretically, ideal filters are not yet commercially available), there is usually a possibility of aliasing occur-



Experiment 1

Experiment 2

ring in the highest frequencies of most FFT spectra. To avoid plotting possibly aliased values, these top lines are usually discarded. Only the lowest 100 lines of a 128 line Spectrum are plotted, the lowest 200 lines of a 256 line Spectrum, the lowest 400 lines of a 512 line Spectrum, etc.

Examples:

(1) To see up to 500 Hz in a Spectrum, the sample rate must be at least 1000 samples per second.

(2) For a sample rate of 32 synchronous samples per revolution, the highest frequency line in a Spectrum will (theoretically) be 16 orders of running speed (16X).

Note that this law says nothing about how many samples are taken or how long it takes to collect all of the samples. Consequently, it doesn't say anything about how many Spectrum lines there will be or what the spacing between the lines will be either.

Law #2: The total sample time governs the resolution

The resolution you get out of an FFT, i.e. the spacing between Spectrum lines, is the reciprocal of the length of time you sample. If you have to resolve or distinguish two separate frequencies which are close together (small Δf), you must sample for a long time (large T).

$$\Delta f = 1/T$$

where Δf = the spacing between frequency lines in Hertz

T = the total sample time in seconds

Look at Experiment 1. A waveform has been printed in yellow ink on a blue background. Since yellow and blue are well-separated frequencies, the waveforms can be quickly distinguished. In Experiment 2, we have changed the blue background to a shade of yellow close in hue to the shade of yellow used for the waveform. It is now more difficult to distinguish the yellow figure on the yellow background, and you will have to look for a longer period of time before you can resolve the waveform.

Examples:

(1) For a total sample time of 0.5 seconds, the frequency lines of the Spectrum will be 2 Hz apart.

(2) To resolve two frequency components which are 6 cpm (0.1 Hz) apart, the signal must be sampled for at least 10 seconds.

(3) For a total sample time of 8 revolutions, the frequency lines of the Spectrum will be spaced 1/8 of an order of running speed apart (1/8X).

Note that this law says nothing about what the sample rate is, or how many samples are collected. Correspondingly, it does not say anything about frequency response, or how many Spectrum lines there will be either.

Law #3: The number of samples governs the number of lines in the Spectrum.

FFTs use complex numbers. The number of frequency domain values you get out of an FFT is the same as the number of time domain samples you put in. Think of it as a Conservation of Numbers principle at work here. Each frequency line from an FFT has a direct or cosine part and a quadrature or sine part. These are called "real" and "imaginary," respectively, in some math books. If you put in 2048 time domain samples, you obtain 2048 frequency domain values — a 1024 line Spectrum where each line has two values, a direct value and a quadrature value. Note that this is equivalent to saying that each Spectrum line has both an amplitude value and a phase value.

For rotating machinery, we almost always plot only the amplitude values from the complex output of an FFT; the phase values are not plotted as part of a Spectrum. These amplitude spectra, therefore, consist of half the number lines as there are time domain samples. The phase values are not discarded; they are simply used elsewhere. Typically, only the phase of synchronous frequency lines (integral multiples of running speed, i.e. 1X, 2X, 3X, etc., or occasionally integral submultiples of running speed, i.e. 1/2X, 1/3X, 1/4X, 2/5X, etc.) are used. The phase of all non-synchronous frequency lines will vary as a function of exactly when the signal is sampled; sampling either a fraction of a second earlier or later can produce wildly different phase values.

Note that this law says nothing about the sample rate, or the total time it takes to collect all of the samples. Correspondingly, it does not say anything about frequency response or frequency resolution either. ►

$$\text{Amplitude value} = \sqrt{\text{“Direct” value}^2 + \text{“Quadrature” value}^2}$$

$$\text{Phase value} = \text{Arctangent}\ \frac{\text{“Quadrature” value}}{\text{“Direct” value}}$$

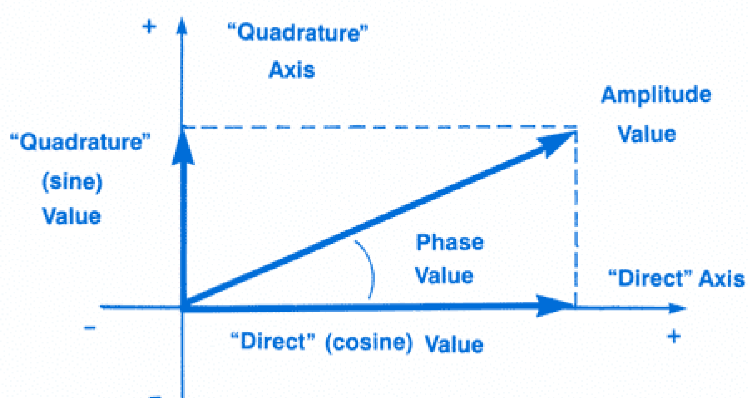


Figure 1
Complex Spectrum Lines

Examples:

These three Laws of the Universe are interdependent. Again, you must specify any two of them to meet your requirements; the third then becomes fixed, and you can't do anything to change it.

(1) If you sample for 0.5 seconds and you want a 3200 line Spectrum (which is a truncated 4096 line Spectrum):

Law #2: $\Delta f = 1/T = 1/0.5$ second $= 2$ Hz.

Law #3: a 4096 line Spectrum requires 8192 samples

$\therefore F_S = 8192$ samples/0.5 seconds $= 16{,}384$ samples/second

$F_{Max} = 16{,}384\ \text{Hz}/2 = 8192$ Hz

$F_{line\ 3200} = 3200$ lines $\times$ 2 Hz/line $= 6400$ Hz

(2) If you have to resolve frequencies 12 cpm (0.2 Hz) apart, and you want an 800 (really 1024) line Spectrum:

Law #2: $T = 1/\Delta f = 1/0.2\ \text{Hz} = 5$ seconds

Law #3: a 1024 line Spectrum requires 2048 samples

$\therefore F_S = 2048$ samples/5 seconds $= 409.6$ samples/second

$F_{Max} = 409.6\ \text{Hz}/2 = 204.8$ Hz

$F_{line\ 800} = 800$ lines $\times$ 0.2 Hz/line $= 160$ Hz

(3) If you have to resolve frequencies 6 cpm (0.1 Hz) apart and the highest frequency you need to see is 180 Hz:

Law #1: $F_S \geq 2 \bullet F_{Max} \geq 2 \bullet 180$ Hz ≥ 360 Hz

Law #2: $T = 1/\Delta f = 1/0.1\ \text{Hz} = 10$ seconds

Therefore, taking 360 samples/second (or more) for 10 seconds = 3600 (or more) samples.

The next higher power of 2 is 4096. 4096 samples will yield a 2048 line Spectrum which will be truncated to 1600 lines. A 1600 line Spectrum with lines 0.1 Hz apart only shows up to 160 Hz, which is not high enough to meet requirements.

The next higher power of 2 is 8192. 8192 samples will yield a 4096 line Spectrum which will be truncated to 3200 lines. A 3200 line Spectrum with lines 0.1 Hz apart will show up to 320 Hz.

(4) If you take 32 samples per revolution for 8 revolutions, you will take a total of 256 samples and get a 128 line Spectrum, which will truncate to 100 lines, and:

$T = 8$ revolutions

$\Delta f = 1/T = 1/8$ revolutions $= 1/8X$

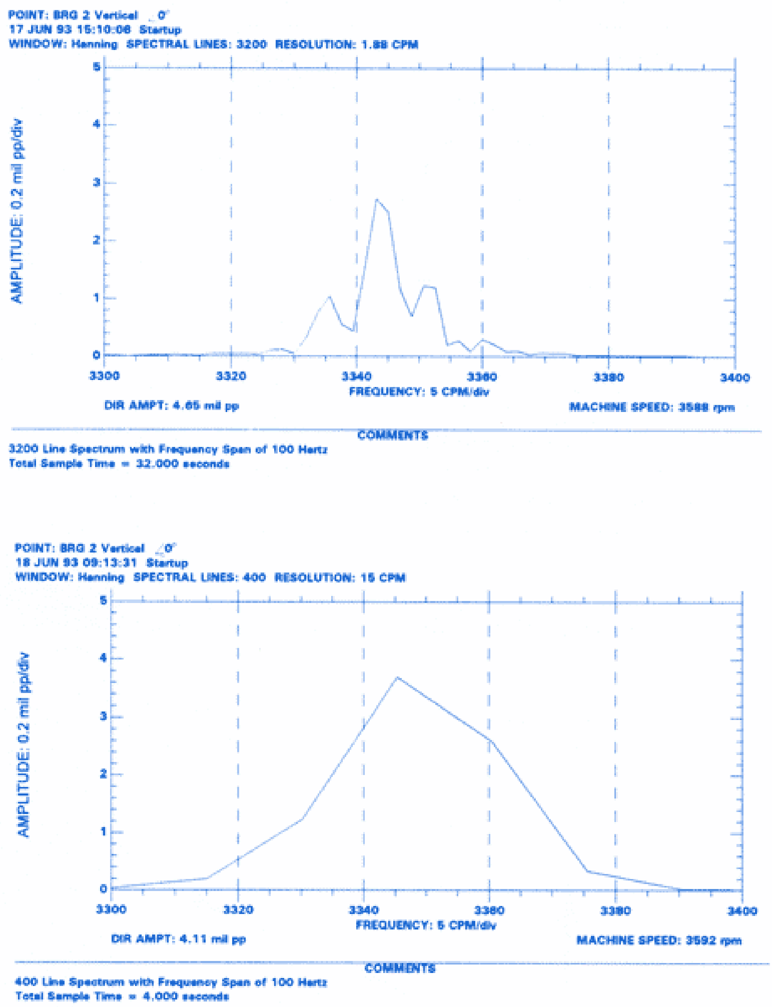
$F_{Max} = 32$ samples per revolution$/2 = 16X$

$F_{line\ 100} = 100$ lines $\times$ 0.125 Hz/line $= 12.5X$

Digital Signal Processing (DSP) "Tricks"

There are also some DSP techniques which can be used to "enhance" an FFT. But be aware, there are always tradeoffs.

Without resampling, the number of Spectrum lines can be increased, and the line spacing can be decreased, pro-



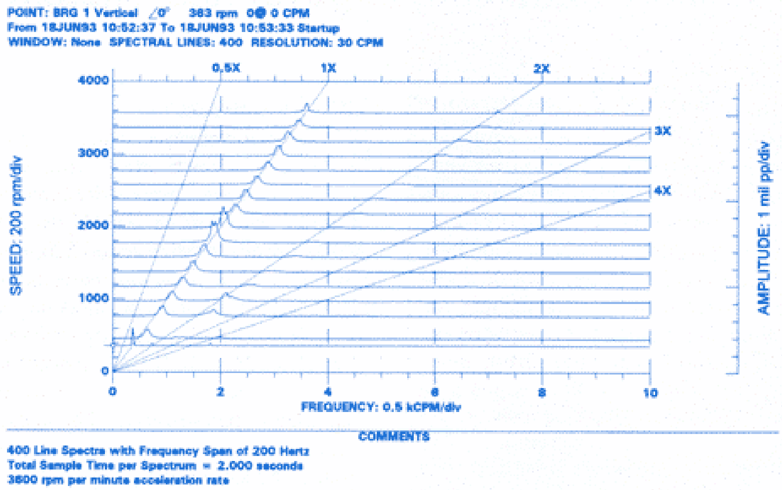
Figures 2 & 3

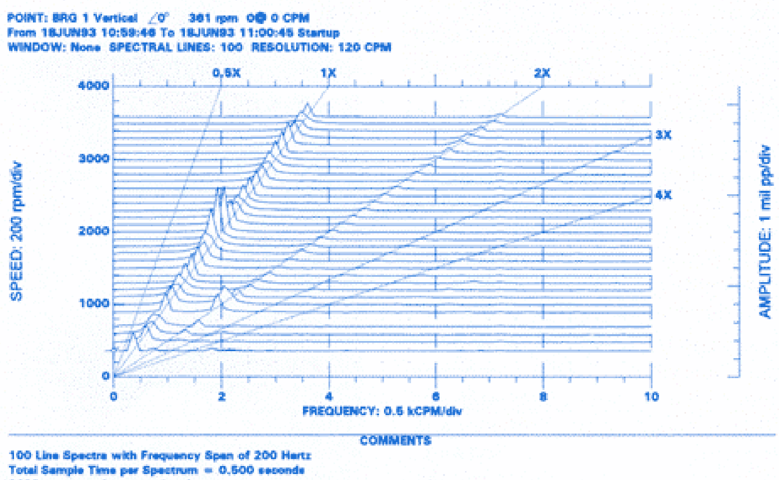
For steady state applications, i.e. when the machine is running at a constant speed, the signal can be sampled over a longer time period. Notice that the 3200 line Spectrum (Figure 1) required 32 seconds to acquire the waveform data. The 400 line Spectrum (Figure 2) only sampled for 4 seconds, and provides inadequate resolution.

ducing an "enhanced" Spectrum. To accomplish this, one major assumption must be made: a set of samples taken over a relatively short period of time must accurately represent a longer term behavior of the signal. For example, you assume that if you were to sample 2, 4, 8, etc., times longer, the signal wouldn't change characteristics. Without getting too technical, one technique (called zero-padding) and the rationale behind it is as follows:

If you sample a signal at a certain rate for a certain period of time, say 2048 samples per second for 0.5 seconds, you will get 1024 samples, a 512 line Spectrum with a high frequency of 1024 Hz and a resolution of 2 Hz. Assuming that this 0.5 second period accurately represents the behavior of the signal over a 1.0 second period, scale the samples (multiply each sample value by an amplitude adjustment factor), put them in the first half of a 2048 sample buffer, fill the last half of the buffer with zeros, then perform the FFT. The resulting Spectrum will have 1024 lines, a high frequency of 1024 Hz and a resolution of 1 Hz.

This same technique will work with other larger buffers, such as ones four times as large. In this example, fill the first 1/4 of the buffer with scaled samples and the last 3/4 with zeros. If you have a buffer which is eight times as big, fill the first 1/8 of the buffer with scaled samples and the last 7/8 with zeros, etc. These "enhanced" spectra will have both high resolution and short sample times.





Figures 4 & 5

For transient applications, i.e. startup/shutdown, the signal should be sampled over as short a time period as possible to avoid "smearing." Smearing occurs when the machine speed changes while the data is being sampled, i.e. the machine speed might be 2000 rpm at the beginning of the sample period and, due to acceleration, be 2200 rpm at the end of the sample period. Comparing Figure 3 and Figure 4, notice that the 100 line Spectrum provides better information than the 400 line Spectrum (due to the shorter sample period).

Information accuracy and quality

Zero-padding changes only the computational line spacing; the effective resolution bandwidth remains the same no matter how many zeros are added (Reference 1). In other words, the information content (the effective resolution bandwidth) is the same with or without the zero padding.

The more accurately the samples taken over a short time period represent what really happened over a longer (but not totally sampled) time period, the more accurate the enhanced Spectrum will be. If the samples taken over a short time period do not accurately represent the signal over the longer period, the enhanced Spectrum will misrepresent what really happened over the longer period. The higher the degree of enhancement, for example, 8 to 1, 16 to 1, 32 to 1, etc., the more likely it is for this misrepresentation to occur.

Sometimes these DSP "tricks" are valid and sometimes they aren't. Universal application of any technique which might be valid only within a limited scope of circumstances can produce poor or misleading results. ■

Reference

1. Robert K. Otnes and Loren Enochson, *Applied Time Series Analysis,* John Wiley & Sons, New York, 1978, p. 235.