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Review of the Concept of Dynamic Coefficients for Fluid Film Journal Bearings

The development of the concept of spring and damping coefficients for journal bearings is briefly reviewed. Methods for computing the coefficients are described, and their use in rotor dynamics calculations (unbalance response, stability) is discussed. The limitations imposed by nonlinearities on the application of the coefficients is illustrated by examples.

Introduction

The idea of representing the dynamic response characteristics of a journal bearing by means of stiffness and damping coefficients originates with Stodola [1] and Hummel [2]. Their aim was to improve the calculation of the critical speed of a rotor by including the flexibility of the bearing oil film. Concurrently, Newkirk [3, 4] described the phenomenon of bearing induced instability, which he called oil whip, and it soon occurred to several investigators that the problem of rotor stability could be related to the properties of the bearing coefficients as can be seen from numerous references in the reference list.

In the initial stage, one of the difficulties was that the only available solution of Reynold's equation was that of Sommerfeld for the infinitely long bearing, sometimes modified by a suitable end leakage factor, and that the only calculations of rotor dynamics was that of the first critical speed, using the methods of either Rayleigh or Stodola.

In the late forties, more advanced rotor dynamics calculation methods were developed, and with the advent of the computer in the fifties, it soon became feasible at little cost to obtain numerical solutions of Reynold's equation and to perform more elaborate rotor calculations.

Even so, the concept of bearing coefficients was not immediately accepted, probably because the load-displacement characteristic of a journal bearing is so evidently non-linear. Experience, however, has demonstrated the practical usefulness of the coefficients, and modern rotor dynamics calculations are firmly based on the concept.

Today the emphasis is on experimental measurements of the bearing coefficients and establishing more uniform agreement with calculated values. The need for some improvements or refinements in the theory seem indicated, but the basic calculation method appears sound.

The appended reference list gives some, but far from all of the papers that mark the development of the concept of bearing coefficients. Little attempt has been made of a historical review, and there is no systematic scheme behind the selection of the papers. The cited works, however, are sufficiently representative to allow compiling a comprehensive list by complementing the present reference list by those of the papers.

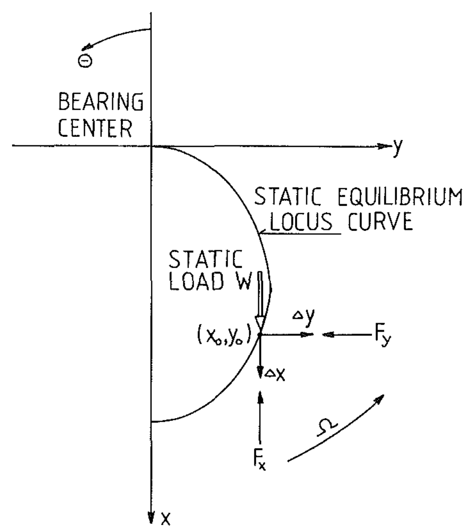

Fig. 1 Coordinate system

Analysis

In an x-y-coordinate system with origin in the bearing center and the x-axis in the static load direction (see Fig. 1), the reaction forces from the lubricant film are given by:

$$\begin{Bmatrix} F_x \\ F_y \end{Bmatrix} = -\int_{-L/2}^{L/2}\int_{\theta_1}^{\theta_2} p \cdot \begin{Bmatrix} \cos\theta \\ \sin\theta \end{Bmatrix} R d\theta dz \qquad (1)$$

where p is the pressure in the film, R is the journal radius, L is the axial length, z is the axial coordinate, and θ is the cir-

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cumferential angular coordinate, measured from the negative x-axis. The film extends from θ_1 to θ_2 where both angles may be functions of z.

The film pressure derives from Reynold's equation:

$$\frac{\partial}{R\partial\theta}\left(\frac{h^3}{12\mu}\frac{\partial p}{R\partial\theta}\right)+\frac{\partial}{\partial z}\left(\frac{h^3}{12\mu}\frac{\partial p}{\partial z}\right)=½\Omega\frac{\partial h}{\partial\theta}+\frac{\partial h}{\partial t} \quad (2)$$

where t is time, μ is the lubricant viscosity, Ω is the journal angular speed of rotation, and h is the film thickness:

$$h=C+x\cos\theta+y\sin\theta \quad (3)$$

C is the radial clearance, and x and y are the coordinates of the journal center.

It appears that the reaction forces are functions of x and y, and of the instantaneous journal center velocities, $\dot{x}$ and $\dot{y}$ ("dot" indicates time derivative). Hence, for small amplitude motions, Δx and Δy, measured from the static equilibrium position (x_0, y_0), a first order Taylor series expansion yields:

$$\begin{aligned}F_x&=F_{xo}+K_{xx}\Delta x+K_{xy}\Delta y+B_{xx}\Delta\dot{x}+B_{xy}\Delta\dot{y}\\F_y&=F_{yo}+K_{yx}\Delta x+K_{yy}\Delta y+B_{yx}\Delta\dot{x}+B_{yy}\Delta\dot{y}\end{aligned} \quad (4)$$

where the coefficients are the partial derivatives evaluated at the equilibrium position:

$$K_{xy}=\left(\frac{\partial F_x}{\partial y}\right)_0 \quad B_{xy}=\left(\frac{\partial F_x}{\partial\dot{y}}\right)_0 \quad (5)$$

and analogously for the remaining coefficients.

Because (x_0, y_0) is the equilibrium position, then $F_{y0}=0$ while F_{x0} equals the static load, W.

In the literature the coefficients are often computed directly by numerical differentiation. While this may be adequate for practical purposes, the inherent numerical inaccuracy of the method can be eliminated by employing a perturbation solution. Thus, equation (3) may be written as:

$$h=h_0+\Delta h \quad (6)$$

where:

$$h_0=C+x_0\cos\theta+y_0\sin\theta \quad (7)$$

$$\Delta h=\Delta x\cos\theta+\Delta y\sin\theta \quad (8)$$

$$\frac{\partial h}{\partial t}=\Delta\dot{x}\cos\theta+\Delta\dot{y}\sin\theta \quad (9)$$

The perturbation in film thickness gives rise to a similar perturbation in the film pressure:

$$p=p_0+\Delta p \quad (10)$$

where:

$$\Delta p=p_x\cdot\Delta x+p_y\cdot\Delta y+p_x'\cdot\Delta\dot{x}+p_y'\cdot\Delta\dot{y} \quad (11)$$

By substituting equation (6) to (11) into Reynold's equation, equation (2), and by retaining first order terms only, five equations are obtained:

$$R\{p_0\}=½\Omega\frac{\partial h_0}{\partial\theta}=½\Omega(-x_0\sin\theta+y_0\cos\theta) \quad (12)$$

$$R\{p_x\}=-3\frac{\cos\theta}{h_0}½\Omega\frac{\partial h_0}{\partial\theta}-3\frac{h_0^3}{12\mu}\frac{\partial p_0}{R\partial\theta}\frac{\partial}{R\partial\theta}\left(\frac{\cos\theta}{h_0}\right)-½\Omega\sin\theta \quad (13)$$

$$R\{p_y\}=-3\frac{\sin\theta}{h_0}½\Omega\frac{\partial h_0}{\partial\theta}-3\frac{h_0^3}{12\mu}\frac{\partial p_0}{R\partial\theta}\frac{\partial}{R\partial\theta}\left(\frac{\sin\theta}{h_0}\right)+½\Omega\cos\theta \quad (14)$$

$$R\{p_x'\}=\cos\theta \quad (15)$$

$$R\{p_y'\}=\sin\theta \quad (16)$$

where the left-hand side operator is:

$$R\{\ \}=\frac{\partial}{R\partial\theta}\left(\frac{h_0^3}{12\mu}\frac{\partial}{R\partial\theta}\right)+\frac{\partial}{\partial z}\left(\frac{h_0^3}{12\mu}\frac{\partial}{\partial z}\right) \quad (17)$$

The boundary conditions are that the pressure is zero at the edges ($p_0=\Delta p=0$ at $z=\pm ½L$) and equals the feeding pressure at any supply grooves.

When film rupture occurs in the divergent film region, the location of the trailing end boundary is governed by the condition of zero pressure gradient, requiring some iterative procedure to determine the actual boundary curve. If (θ_0, z_0) is a point on the curve, the pressure in a neighboring point is, to the first order:

$$\begin{aligned}p(\theta_0+\Delta\theta,z_0+\Delta z)&=p(\theta_0,z_0)+\left(\frac{\partial p}{\partial\theta}\right)_0\Delta\theta+\left(\frac{\partial p}{\partial z}\right)_0\Delta z\\&\cong p_0(\theta_0,z_0)+\Delta p(\theta_0,z_0)+\left(\frac{\partial p_0}{\partial\theta}\right)_0\Delta\theta+\left(\frac{\partial p_0}{\partial z}\right)_0\Delta z\end{aligned} \quad (18)$$

With $p_0(\theta_0, z_0)$, $(\partial p_0/\partial\theta)_0$, $(\partial p_0/\partial z)_0$ being zero, and by assuming that the pressure on the boundary, $p(\theta_0+\Delta\theta,z_0+\Delta z)$, is zero also under dynamic conditions then the boundary condition for the perturbed pressure is that Δp is zero on the statically determined curve. In case of film rupture (film contraction) ahead of the leading edge of the film, the location of the boundary curve is governed by flow equilibrium such that the pressure gradient is not zero. Even so, the condition of Δp being zero on the boundary curve is sufficiently accurate for most purposes.

By combining equations (1), (4), (10), and (11) it is found that:

$$\left.\begin{matrix}K_{xx}\\K_{yx}\end{matrix}\right\}=-\int_{-L/2}^{L/2}\int_{\theta_1}^{\theta_2}p_x\left\{\begin{matrix}\cos\theta\\\sin\theta\end{matrix}\right\}Rd\theta dz \quad (19)$$

and analogously for the remaining coefficients.

Because they derive from self-adjoint operators, the cross-coupling damping coefficients are equal:

$$B_{yx}=B_{xy} \quad (20)$$

The damping matrix, therefore, is symmetric and possesses principal directions. This is not true for the stiffness matrix wh ere the cross-coupling stiffness coefficients are unequal:

$$K_{yx}\neq K_{xy} \quad (21)$$

Thus, the stiffness matrix is nonconservative as is readily demonstrated by considering a closed whirl orbit with angular whirl frequency ω:

$$\begin{aligned}\Delta x&=|\Delta x|\cos(\omega t+\phi_x)\\\Delta y&=|\Delta y|\sin(\omega t+\phi_y)\end{aligned} \quad (22)$$

where ϕ_x and ϕ_y are phase angles. The energy dissipated over one cycle is:

$$\begin{aligned}E_{\text{diss}}=\pi\omega(&B_{xx}|\Delta x|^2+B_{yy}|\Delta y|^2\\&-(B_{xy}+B_{yx})|\Delta x|\cdot|\Delta y|\sin(\phi_x-\phi_y)\\&-\pi(K_{xy}-K_{yx})|\Delta x|\cdot|\Delta y|\cos(\phi_x-\phi_y))\end{aligned} \quad (23)$$

With K_{xy} positive, the cross-coupling stiffness coefficients may provide negative damping or, in other words, they can supply energy to the motion. The last term may also be written as:

$$\pi(K_{xy}-K_{yx})(-|\Delta x_f|^2+|\Delta x_b|^2) \quad (24)$$

where the elliptical whirl orbit has been decomposed into two circular orbits, one with forward whirl direction and amplitude $|\Delta x_f|$, and one with backward whirl direction and amplitude $|\Delta x_b|$. Thus it is the forward whirl component which is responsible for the negative damping.

It is customary to analyze the stability of a bearing by assigning a mass M to the journal such that the equations of motion become:

$$M\Delta\ddot{x}+B_{xx}\Delta\dot{x}+K_{xx}\Delta x+B_{xy}\Delta\dot{y}+K_{xy}\Delta y=0$$
$$M\Delta\ddot{y}+B_{yy}\Delta\dot{y}+K_{yy}\Delta y+B_{yx}\Delta\dot{x}+K_{yx}\Delta x=0 \quad (25)$$

These equations admit of solutions for Δx and Δy of the form e^{st} where:

$$s=\lambda+i\omega \quad (i=\sqrt{-1}) \quad (26)$$

For a nontrivial solution, the eigenvalues s are the roots of the determinant of the coefficient matrix. At the threshold of instability, λ is zero whereby equation (25) takes on the form:

$$\begin{Bmatrix} (Z_{xx}-\omega^2 M) & Z_{xy} \\ Z_{yx} & (Z_{yy}-\omega^2 M) \end{Bmatrix}\begin{Bmatrix} \Delta x \\ \Delta y \end{Bmatrix}=0 \quad (27)$$

where:

$$Z_{xx}=K_{xx}+i\omega B_{xx} \quad (28)$$

and similarly for the other impedances.

For a chosen static equilibrium position (Sommerfeld number), the values of the coefficients are given, and setting the determinant equal to zero determines that value of M, called M_{crit}, which makes the specified equilibrium position unstable. The solutions for M_{crit} and the associated whirl frequency, $\omega=\omega_0$, are:

$$\omega_0^2 M_{crit}=\frac{K_{xx}B_{yy}+K_{yy}B_{xx}-K_{xy}B_{yx}-K_{yx}B_{xy}}{B_{xx}+B_{yy}}=\kappa_0 \quad (29)$$

$$\omega_0^2=\frac{(K_{xx}-\kappa_0)(K_{yy}-\kappa_0)-K_{xy}K_{yx}}{B_{xx}B_{yy}-B_{xy}B_{yx}} \quad (30)$$

In order to establish on which side of the threshold the bearing becomes unstable, the determinant, Δ, may be expanded around the threshold value as:

$$\Delta\cong\Delta_0+\left(\frac{\partial\Delta}{i\partial\omega}\right)_0\cdot ds+\left(\frac{\partial\Delta}{\partial M}\right)_0\cdot dM \quad (31)$$

With Δ and Δ_0 equal to zero, the equation may be solved:

$$\left(\frac{ds}{dM}\right)_0=\left(\frac{d\lambda}{dM}\right)_0+i\left(\frac{d\omega}{dM}\right)_0=i\left(\frac{\partial\Delta}{\partial\omega}\right)_0\Big/\left(\frac{\partial\Delta}{\partial M}\right)_0 \quad (32)$$

from which:

$$\left(\frac{d\lambda}{dM}\right)_0=\frac{\frac{1}{2}\omega_0^6(B_{xx}+B_{yy})(B_{xx}B_{yy}-B_{xy}B_{yx})}{\omega_0^2(B_{xx}+B_{yy})^2\kappa_0^2+(K_{xx}K_{yy}-K_{xy}K_{yx}-\kappa_0^2)^2} \quad (33)$$

This value is positive which means that an increase, dM, in journal mass above the critical value results in a positive value of λ or, in other words, causes the bearing to become unstable.

In equation (27) the term $-\omega^2 M$ represents the rotor impedance at the journal. It may be thought of as being opposed by some equivalent bearing impedance Z which, then, is a solution to the determinantal equation:

$$Z=\tfrac{1}{2}(Z_{xx}+Z_{yy})\pm\sqrt{\frac{1}{4}(Z_{xx}-Z_{yy})^2+Z_{xy}Z_{yx}}$$

$$=\begin{cases} Z_{max}=K_{max}+i\omega B_{max} \\ Z_{min}=K_{min}+i\omega B_{min} \end{cases} \quad (34)$$

The effective stiffnesses, K_{max} and K_{min}, and the effective damping coefficients, B_{max} and B_{min}, are of course functions of frequency. For a plain journal bearing B_{max} is always positive, while B_{min} is negative for frequencies smaller than ω_0 (for $\omega=\omega_0$, B_{min} equals zero and K_{min} equals κ_0). Hence, if the lowest natural frequency of the system, ω_{nat}, is less than ω_0, self-excited whirl can be sustained. The speed, Ω_0, at which instability sets in, may then be expressed as:

$$\Omega_0=\frac{\omega_{nat}}{(\omega_0/\Omega)_0} \quad (35)$$

A typical value for the whirl frequency ratio, $(\omega_0/\Omega)_0$, is around 1/2 from which stems the rule-of-thumb that a rotor becomes unstable when the speed reaches twice the first critical speed ($\sim\omega_{nat}$).

Discussion

The preceding stability analysis applies only to a rigid rotor which is symmetric and supported in two identical bearings. Hence, the results are of limited usefulness, and in general it can be quite misleading to rank various bearing types for stability on the basis of the critical mass.

As seen from equation (29), the critical mass value is a measure of the stiffness of the bearing, but an increase in bearing stiffness does not ensure that the lowest natural frequency is raised to improve the instability threshold speed.

Similarly, equation (30) suggests that bearings with a low whirl frequency ratio may be more stable. For a flexible rotor, however, it is found that it is the whirl frequency itself, equal to the lowest natural frequency, which stays unchanged as the rotor becomes unstable, not the whirl frequency ratio.

Furthermore it should be noted that associated with the calculated instability threshold is a characteristic whirl orbit, obtained as the solution for $\Delta y/\Delta x$ from equation (27) with the determinant being zero. For a flexible, non-symmetric rotor, the modeshape will not conform to the characteristic orbit, and thereby the effective stiffness and damping stability properties of the bearing change.

For a practical rotor there is no real substitute for a stability calculation in which the rotor, with its bearings represented by their coefficients, is modeled as close to the actual design as feasible. Such calculations are today standard procedure when designing high speed machinery. The results are in satisfactory agreement with experience, considering the uncertainties and often poorly established tolerances on bearing geometry, clearance, and pedestal stiffness.

A different type of calculation, but with the same rotor-bearing-system model, is the one for the response to mass unbalance. It is used to check the unbalance sensitivity of the rotor and, also, to determine the damped critical speeds.

It is inherent in the definition of the bearing stiffness and damping coefficients that, mathematically, they are only valid for infinitesimal amplitudes. Comparisons with exact solutions, however, prove that for practical purposes the coefficients may be employed for amplitudes as large as, say, forty percent of the clearance.

An example is shown in FIg. 2 which is for a two-axial groove journal bearing with a length-to-diameter ratio of ½. The journal is forced to whirl synchronously by a rotating unbalance force, equal to half the static load, while the inertia of the journal is ignored.

The elliptical whirl orbit, calculated on the basis of the eight coefficients belonging to the indicated static equilibrium position, is compared to the whirl orbit obtained from a true nonlinear solution. It is found that the amplitudes, whith a maximum value of 38 percent of the clearance, are in close agreement, but, as expected, there are also deviations.

First, the center of the orbit moves closer to the bearing center to satisfy overall dynamic equilibrium:

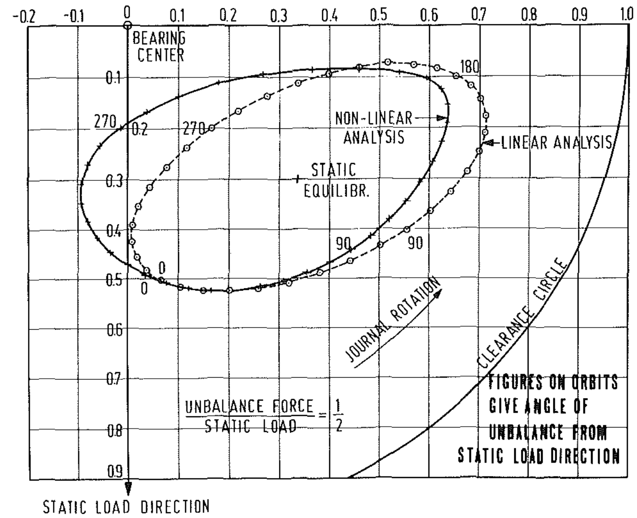

Fig. 2 Unbalance whirl orbits. Comparison between linear and nonlinear analysis.

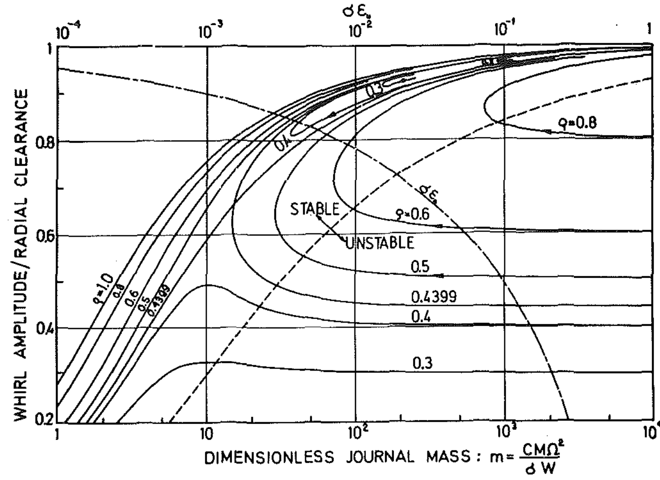

Fig. 3 Circular unbalance whirl orbits for unloaded, short journal bearing with rigid rotor

$$\int_{\text{orbit}} F_x ds = W \quad \int_{\text{orbit}} F_y ds = 0 \tag{36}$$

where s is the arc length along the path of the orbit. Second, the shift in orbit center is accompanied by a change in orbit orientation which affects the agreement in phase angle. Finally, it is evident that the non-linear orbit contains higher order harmonics which of course are absent in the linear solution.

It should be emphasized, however, that in the matter of greatest importance, namely the size of the amplitude, the agreement is close. This holds true in the range of interest in practice where amplitudes larger than, say, 40 percent of the clearance would rarely be tolerated.

For even larger amplitudes, the various phenomena associated with non-linear vibrations become evident. As an example, Fig. 3 illustrates the existence of multiple solutions in certain operating regions. The diagram is computed for the idealized case of an unloaded, infinitely short journal bearing where the journal, with mass M, whirls synchronously in a circular orbit whose radius, normalized with respect to the radial clearance C, is given by the ordinate.

The curves in the diagram are labeled by the dimensionless unbalance mass eccentricity ρ, normalized by C. The abscissa gives the dimensionless journal mass $m = CM\Omega^2/\sigma W$ where σ is the short bearing Sommerfeld number:

$$\sigma = \frac{\mu \Omega R L}{W} \left(\frac{R}{C}\right)^2 \left(\frac{L}{D}\right)^2 \tag{37}$$

Hence, for a particular rotor and bearing, the abscissa is simply proportional to the rotational speed.

It is seen that, for some given mass unbalance, there will be a speed beyond which more than one solution is possible and where jump phenomena may occur. The linearized solution with stiffness and damping coefficients agrees well with the small amplitude orbit, but it is of course unable to predict the concurrent orbits with larger amplitudes.

Similarly the previously discussed stability analysis, based on the liniearized coefficients, is concerned only with the stability of the static equilibrium and the inception of whirl, and it can not be used to study whether that whirl orbit may actually be contained and stable (limit cycle). Furthermore, while the linear analysis predicts that an unloaded journal bearing is inherently unstable, the stability curve in Fig. 3 demonstrates that stable synchronous whirl orbits may exist for ρ greater than 0.13 or, in other words, the bearing can be "stabilized" by adding unbalance.

The nonlinear behaviour, however, is rarely of much concern in practice, and for practical purposes the instability threshold computed on the basis of the linearized coefficients should be adopted as the design criterion. Past the threshold speed, self-excited whirl takes place in one form or another, and although numerous cases are on record of safe operation in this region, at least as many cases of uncontrollable whirl can be cited.

The threshold based on the coefficients proves in general, therefore, to be a conservative estimate. There remain, however, some basic questions about applying Reynold's equation. The major problem is the physics of film rupture which has yet to be clarified, and which makes uncertain what the proper boundary conditions are under dynamic load. As a curiosity, another example is the question of the thermal conditions prevailing under synchronous whirl when the orbit encircles the bearing center.

Finally, the four stiffness coefficients can be derived directly from the static equilibrium locus curve. It is unfortunate, therefore, that experimental measurements of the curve all too frequently deviate appreciably from the theoretical curve. The measurements are admittedly not easy to make, and the experimental tolerances are difficult to ascertain, but enough results have been published to indicate that the discrepancies are real.

Under those conditions, experimental measurements of the coefficients (typically by applying a vibratory force and measure the resulting response) can not be expected to agree well with the theoretical values. The experiments are no longer a test of the validity of the dynamic analysis; rather, they become indirect evidence of discrepancies in the more fundamental solution of Reynold's equation for static conditions.

Conclusion

The concept of stiffness and damping coefficients for journal bearings has proven very fruitful, and modern rotor dynamics calculations for unbalance response, damped natural frequencies, and stability are based on this concept. The theoretical limitation of small amplitudes is of little importance in practice, but the analysis still needs improvements, for example in refinements in the model for film rupture.

In recent years, extensive experimental programs have been carried out to measure the bearing coefficients, not just in the laboratory, but also for large industrial bearings. While many results show good agreement with the theoretical values, cases are still reported of discrepancies that should be resolved.

The question of the influence from thermal and elastic deformations needs further consideration as do also the practical problems of manufacturing and operating tolerances on bearing geometry, clearance, and lubricant viscosity.

References

1 Stodola, A., "Kristische Wellenstörung infolge der Nachgiebigkeit des Oelpolsters im Lager," *Schweizerische Bauzeiting*, Vol. 85, 1925, pp. 265-266.

2 Hummel, C., "Kristische Drehzahlen als Folge der Nachgiebigkeit des Schmiermittels im Lager," VDI-Forschungsheft 287, 1926.

3 Newkirk, B. L., "Shaft Whipping," *General Electric Review*, 1924, p. 169.

4 Newkirk, B. L., and Taylor, H. D., "Shaft Whipping due to Oil Action in Journal Bearings," *General Electric Review*, 1925, pp. 559-568.

5 Robertson, D., "Whirling of a Journal in a Sleeve Bearing," *Philosophical Magazine*, Series 7, Vol. 15, 1933, pp. 113-130.

6 Hagg, A. C., "The Influence of Oil-Film Journal Bearings on the Stability of Rotating Machines," ASME *Journal of Applied Mechanics*, Vol. 13, 1946, pp. A-211 to A-220.

7 Hagg, A. C., and Warner, P. C., "Oil Whip of Flexible Rotors," Trans. ASME, Vol. 75, No. 7, 1953, pp. 1339-1344.

8 Hagg, A. C., and Sankey, G. O., "Some Dynamic Properties of Oil-Film Journal Bearings with Reference to the Unbalance Vibration of Rotors," ASME *Journal of Applied Mechanics*, Vol. 23, 1956, pp. 302-306.

9 Hagg, A. C., and Sankey, G. O., "Elastic and Damping Properties of Oil-Film Journal Bearings for Application to Unbalance Vibration Calculations," ASME *Journal of Applied Mechanics*, Vol. 25, 1958, p. 141.

10 Poritsky, H., "Contribution to the Theory of Oil Whip," *Trans. ASME*, Vol. 75, 1953, pp. 1153-1161.

11 Pinkus, O., "Note on Oil Whip," ASME *Journal of Applied Mechanics*, Vol. 20, 1953, pp. 450-451.

12 Pestel, E., "Beitrag zur Ermittlung der Hydrodynamischen Dämpfungs und Federeigenschaften von Gleitlagren," *Ingenieur-Archiv*, Vol. XXII, No. 3, 1954, pp. 147-155.

13 Cameron, A., "Oil Whirl in Bearings-Theoretical Deduction of a Further Criterion," *Engineering*, London, Feb. 25, 1955, pp. 237-239.

14 Korovchinskii, M. V., "Stability of Position of Equilibrium of a Journal on an Oil Film," *Friction and Wear in Machinery*, Vol. 11, 1956, pp. 248-305.

15 Tondl, A., "The Motion of a Journal in a Bearing in the Unstable Region of Equilibrium Position of the Center of the Journal," Ninth International Congress of Applied Mechanics, Vol. V, 1957.

16 Tondl, A., *Some Problems of Rotor Dynamics*, Publishing House of the Czechoslovak Academy of Sciences, Prague, 1965.

17 Hori, Y., "A Theory of Oil Whip," ASME *Journal of Applied Mechanics*, Vol. 26, 1959, pp. 189-198.

18 Krämer, E., "Der Einfluss des Ölfilms von Gleitlagern auf die Schwingungen von Maschinenwellen," *VDI-Berichte*, Vol. 35, 1959.

19 Sternlicht, B., "Elastic and Damping Properties of Cylindrical Journal Bearings," ASME *Journal of Basic Engineering*, Vol. 81, 1959, pp. 101-108.

20 Capriz, G., "On the Vibrations of Shafts Rotating on Lubricated Bearings," *Annali de Matematica Pura e Applicata*, IV, Ser. 50, 1960, p. 223.

21 Lund, J. W., and Sternlicht, B., "Rotor-Bearing Dynamics with Empasis on Attenuation," ASME *Journal of Basic Engineering*, Vol. 84, No. 4, 1962, pp. 491-502.

22 Lund, J. W., "Spring and Damping Coefficients for the Tilting Pad Journal Bearing," *Trans. ASLE*, Vol. 7, 1964, pp. 342-352.

23 Lund, J. W., "Rotor-Bearing Dynamics Design Technology. Part III: Design Handbook for Fluid Film Type Bearings," Report AFAPL-TR-65-45, Part III, Air Force Aero Propulsion Laboratory, Wright-Patterson Air Force Base, Ohio, May 1965.

24 Holmes, R., "Oil-Whirl Characteristics of a Rigid Rotor in 360° Journal Bearings," *Proced. of the Institution of Mechanical Engineers*, Vol. 177, No. 11, 1963, pp. 291-307.

25 Holmes, R., "Non-Linear Performance of Turbine Bearings," *Journal of Mechanical Engineering Science*, Vol. 12, No. 6, 1970, pp. 377-379.

26 Someya, T., "Stability of a Balanced Shaft Running in Cylindrical Journal Bearings," *Proc. of the Second Lubrication and Wear Conference*, The Institution of Mechanical Engineers, London, 1964, Paper No. 21.

27 Warner, P. C., "Static and Dynamic Properties of Partial Journal Bearings," ASME *Journal of Basic Engineering*, Vol. 85, 1963, pp. 247-257.

28 Warner, P. C., and Thoman, R. J., "The Effect of the 150-Degree Partial Bearing on Rotor-Unbalance Vibration," ASME *Journal of Basic Engineering*, Vol. 86, 1964, pp. 337-347.

29 Glienicke, J., "Experimental Investigation of the Stiffness and Damping Coefficients of Turbine Bearings," *Proced. of the Institution of Mechanical Engineers*, Vol. 181, Part 3B, 1966-67, pp. 116-129.

30 Orcutt, F. K., and Arwas, E. B., "The Steady-State and Dynamic Characteristics of a Full Circular Bearing and a Partial Arc Bearing in the Laminar and Turbulent Flow Regimes," ASME JOURNAL OF LUBRICATION TECHNOLOGY, Vol. 89, 1967, p. 143-153.

31 Lund, J. W., "Calculation of Stiffness and Damping Properties of Gas Bearings," ASME JOURNAL OF LUBRICATION TECHNOLOGY, Vol. 90, 1968, pp. 793-803.

32 Smith, D. M., *Journal Bearings in Turbomachinery*, Chapman and Hall Ltd., London, 1969.

33 Ota, H., and Inagaki, M., "Elastic and Damping Properties of Journal Bearins," *Bulletin of the JSME*, Vol. 17, No. 108, 1974, pp. 693-700.

34 Morton, P. G., "The Derivation of Bearing Characteristics by Means of Transient Excitation Applied Directly to a Rotating Shaft," Dynamics of Rotors, IUTAM Symposium 1974, Lyngby, Denmark. Springer-Verlag 1975, pp. 350-379.

35 Malcher, L., "Die Federungs- und Dämpfungseigenschaften von Gleitlagern für Turbomaschinen," Dissertation, Fakultät für Maschinenbau, Technischen Hochschule Karlsruhe, 1975.

36 Reinhardt, E., and Lund, J. W., "The Influence of Fluid Inertia on the Dynamic Properties of Journal Bearings," ASME JOURNAL OF LUBRICATION TECHNOLOGY, Vol. 97, 1975, pp. 159-167.

37 Lund, J. W., and Thomsen, K. K., "A Calculation Method and Data for the Dynamic Coefficients of Oil-Lubricated Journal Bearings," *Topics in Fluid Film Bearing and Rotor Bearing System Design and Optimization*, ASME, 1978, pp. 1-28.

38 Kirk, R. G., "The Influence of Manufacturing Tolerances on Multi-Lobe Bearing Performance in Turbomachinery," ibid., pp. 108-129.

39 Hisa, S., Matsuura, T., and Someya, T., "Experiments on the Dynamic Characteristics of Large Scale Journal Bearings," *Proced. Second International Conf. on Vibrations in Rotating Machinery*, Cambridge, England, The Institution of Mechanical Engineers, 1980, pp. 223-230.

40 Nordmann, R., and Schollhorn, K., "Identification of Stiffness and Damping Coefficients of Journal Bearings by Means of the Impact Method," ibid., pp. 231-238.

41 Diana, G., Borgese, D., and Dufour, A., "Experimental and Analytical Research on a Full Scale Turbine Journal Bearing," ibid., pp. 309-314.

42 Hashimoto, H., Wada, S., and Marukawa, T., "Performance Characteristics of Large Scale Tilting-Pad Journal Bearings," *Bulletin of the JSME*, Vol. 28, No. 242, 1985, pp. 1761-1767.