DME3051

Identification of rotational damping coefficients / Viscosity measurement

Instructor: Prof. Keun Ryu

1. Identification of rotational damping coefficients

Taken from Mechanical Vibrations by S. S. Rao

2.4 Response of First-Order Systems and Time Constant

Consider a turbine rotor mounted in bearings as shown in Fig. 2.17(a). The viscous fluid (lubricant) in the bearings offers viscous damping torque during the rotation of the turbine rotor. Assuming the mass moment of inertia of the rotor about the axis of rotation as J and the rotational damping constant of the bearings as c_t, the application of Newton's second law of motion yields the equation of motion of the rotor as

$$J\dot{\omega} + c_t\omega = 0 \tag{2.47}$$

where ω is the angular velocity of the rotor, $\dot{\omega} = \frac{d\omega}{dt}$ is the time rate of change of the angular velocity, and the external torque applied to the system is assumed to be zero. We assume the initial angular velocity, $\omega(t = 0) = \omega_0$, as the input and the angular velocity of the rotor as the output of the system. Note that the angular velocity, instead of the angular displacement, is considered as the output in order to obtain the equation of motion as a first order differential equation.

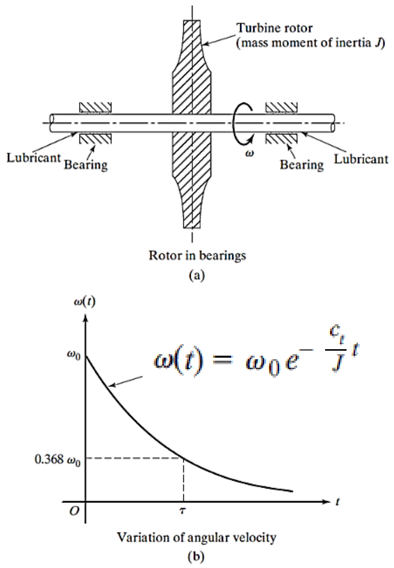

FIGURE 2.17

The solution of the equation of motion of the rotor, Eq. (2.47), can be found by assuming the trial solution as

$$\omega(t) = Ae^{st} \tag{2.48}$$

where A and s are unknown constants. By using the initial condition, $\omega(t = 0) = \omega_0$, Eq. (2.48) can be written as

$$\omega(t) = \omega_0 e^{st} \tag{2.49}$$

By substituting Eq. (2.49) into Eq. (2.47), we obtain

$$\omega_0 e^{st}(Js + c_t) = 0 \tag{2.50}$$

Since $\omega_0 = 0$ leads to "no motion" of the rotor, we assume $\omega_0 \neq 0$ and Eq. (2.50) can be satisfied only if

$$Js + c_t = 0 \tag{2.51}$$

Equation (2.51) is known as the characteristic equation which yields $s = -\frac{c_t}{J}$. Thus the solution, Eq. (2.49), becomes

$$\omega(t) = \omega_0 e^{-\frac{c_t}{J}t} \tag{2.52}$$

The variation of the angular velocity, given by Eq. (2.52), with time is shown in Fig. 2.17(b). The curve starts at ω_0, decays and approaches zero as t increases without limit. In dealing with exponentially decaying responses, such as the one given by Eq. (2.52), it is convenient to describe the response in terms of a quantity known as the *time constant* (τ). The time constant is defined as the value of time which makes the exponent in Eq. (2.52) equal to -1. Because the exponent of Eq. (2.52) is known to be $-\frac{c_t}{J}t$, the time constant will be equal to

$$\tau = \frac{J}{c_t} \tag{2.53}$$

so that Eq. (2.52) gives, for $t = \tau$,

$$\omega(t) = \omega_0 e^{-\frac{c_t}{J}\tau} = \omega_0 e^{-1} = 0.368\omega_0 \tag{2.54}$$

Thus the response reduces to 0.368 times its initial value at a time equal to the time constant of the system.

Taken from Hybrid Flexure Pivot-Tilting Pad Gas Bearings: Analysis and Experimental Validation by Luis San Andrés

The test data show that the decay of shaft speed is of exponential type, i.e., due solely to viscous drag effects, for most of the operating speed range. Thus, a simple model of the form

$$\Omega = \Omega_o e^{-t/\tau}, \quad \tau = I_p/(2C_{\theta\theta}) \tag{25}$$

where τ is the system time constant and $C_{\theta\theta}$ is a rotational viscous damping coefficient derived from the drag torque in the bearings

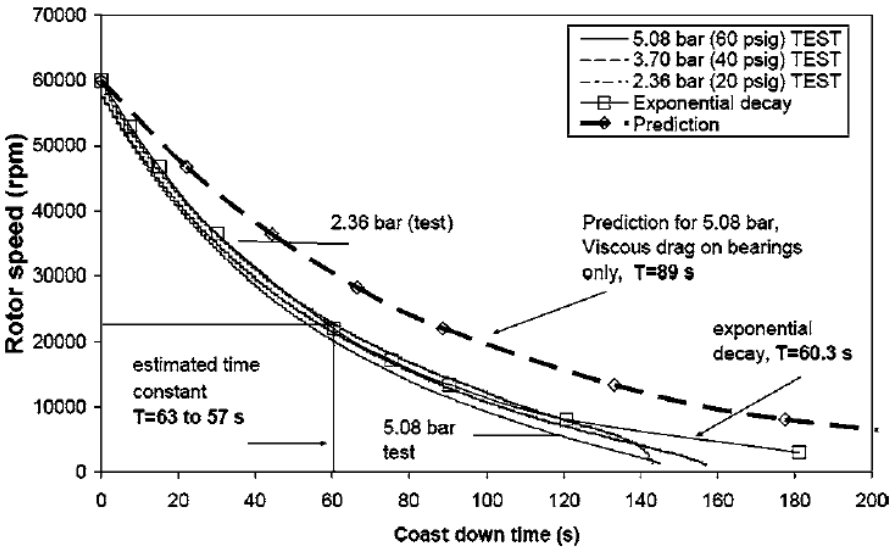

Fig. 11 Recorded coast-down rotor speed versus time for three feed pressures (2.36, 3.70, and 5.08 bar). Estimation of system time constant and speed coast-down predictions based on drag from bearings only.

($C_{\theta\theta}$=Torque/Ω) and windage effects from the motor armature. The experimental time constant for the system ranges from 63 s to 57 s as the supply pressure decreases (average value 60.3 s), while the estimation equals 89 s, as derived from predictions of the drag torque from the gas bearings (alone). The difference is certainly due to the not-quantified drag in the motor. The time constant estimation serves to validate indirectly the prediction of the drag torque in the bearings. A calculated bearing drag friction coefficient, f=Torque/(RW), is proportional to shaft speed. This coefficient ranges from 0.007 to 0.01 as the supply pressure increases. Thus, the tested air bearings do offer nearly negligible friction against rotation.

3. Estimation of Viscosity

Taken from Applied Tribology: Bearing Design and Lubrication by Michael M. Khonsari, E. Richard Booser

CAPILLARY TUBE VISCOMETER

This device consists simply of a small fluid reservoir of height h connected to which is a capillary tube in the vertical position whose length and diameter are known (Figure 5.7). Volumetric flow rate in the capillary is given by the following general relation for a circular flow passage:

$$q = \frac{\pi R^4}{8\mu} \frac{\Delta P}{L} \tag{5.29}$$

where the pressure drop is

$$\Delta P = P_1 - \rho g - (P_a - \rho g) = P_1 - P_a \tag{5.30}$$

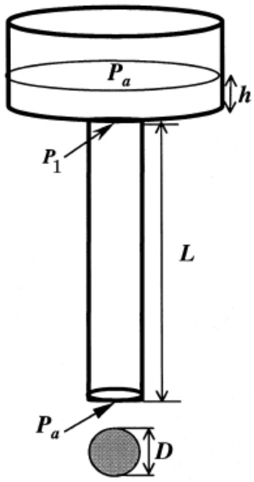

Figure 5.7 Capillary viscometer

But $P_1 = p_a + \rho g h$. Therefore

$$\Delta P = \rho g h \tag{5.31}$$

The flow rate equation becomes

$$q = \frac{\pi R^4}{8\mu} \frac{\rho g h}{L} \tag{5.32}$$

Solving for viscosity yields

$$\mu = \frac{\pi R^4 h}{8L} \frac{\rho g}{q} \tag{5.33}$$