DME3051 Mechanical Design
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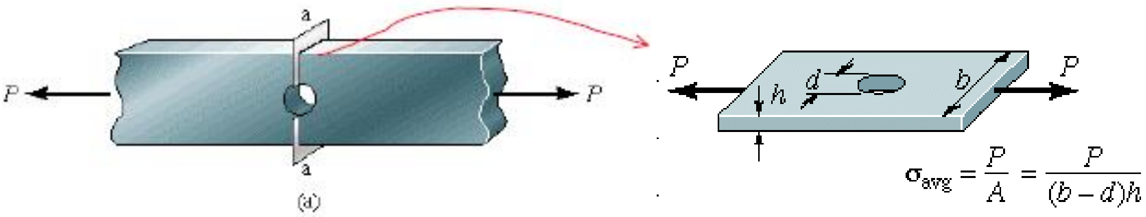
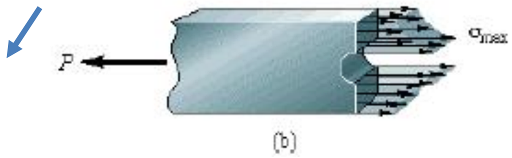
Date: Apr. 20
Note 3: Fatigue Failure

Stress Concentration

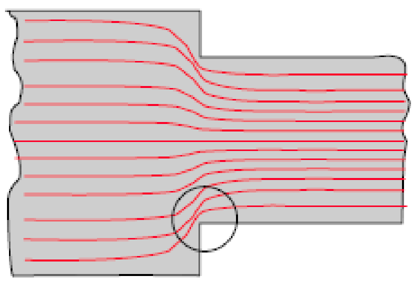
- Stresses at or near a discontinuity such as a hole in a plate are higher than if the discontinuity does not exist.
- Elementary stress equations do not apply in stress concentrations
- Effect of Geometry: The discontinuity geometry has a significant effect on the stress distribution around it.



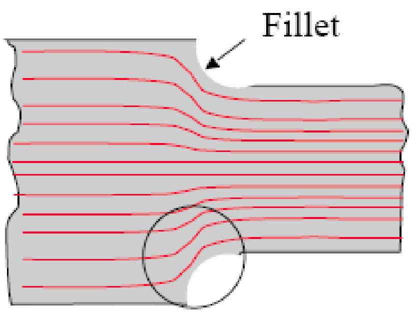


$$\sigma_{Max} = K_t \times \sigma_{avg}$$

where K_t is the (static) stress concentration factor.



Abrupt change
Stress "flow lines" crowd
High stress concentration



Smoother change
"Flow lines" less crowded
Lower stress concentration

Stress concentration occurs at transition of cross sections. The more abrupt the transition, the higher are the stress concentrations.

Stress Concentration Factor K_t

- K_t is difficult to calculate and it is usually determined by some experimental technique (photoelasticanalysis of a plastic model or by numerical simulation of the stress field).
- The values of K_t can be found published in Charts, See your text book.
- The values of K_t are geometric properties.
- K_t is very important in brittle materials.

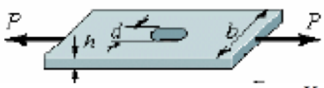
- In ductile materials, K_t is very important in fatigue calculations. It must be taken into account if safety is critical.

Photoelasticity: Photoelasticityis a visual method for viewing the full field stress distribution in a photoelasticmaterial. When a photoelasticmaterial is strained and viewed with a polariscope, distinctive colored fringe patterns are seen. Interpretation of the pattern reveals the overall strain distribution.

Example: Stress concentration for a plate with a hole

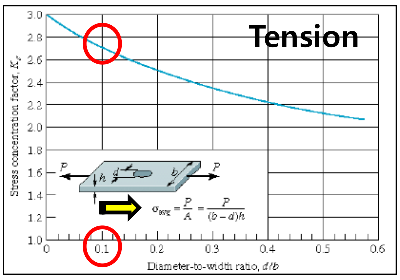
A 50mm wide , 5mm high rectangular plate has a 5mm diameter central hole. The allowable stress due to applying a tensile force is 700MPa. Find (a) the maximum tensile force that can be applied; (b) the maximum bending moment that can be applied; (c) the maximum tensile force and bending moment if the hole if there is no-hole. Compare results.

Solution $\frac{d}{b} = \frac{5}{50} = 0.1$ $\quad Area = A = (b-d)h = 0.225 \times 10^{-3}\, m^2$



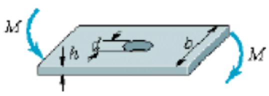
From the Figure

$K_t = 2.70 \quad P_{Max} = \frac{\sigma_{Allowable} A}{K_t} = 58.33kN$

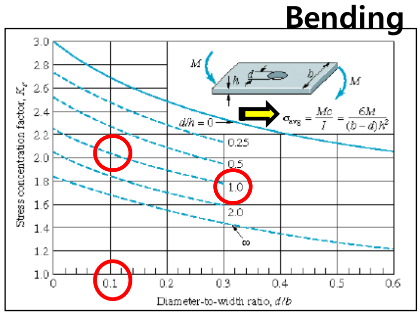


Without a hole $\quad Area = bh = 0.25 \times 10^{-3}\, m^2$

$P_{Max} = \sigma_{Allowable} \times A = 175kN$



Bending

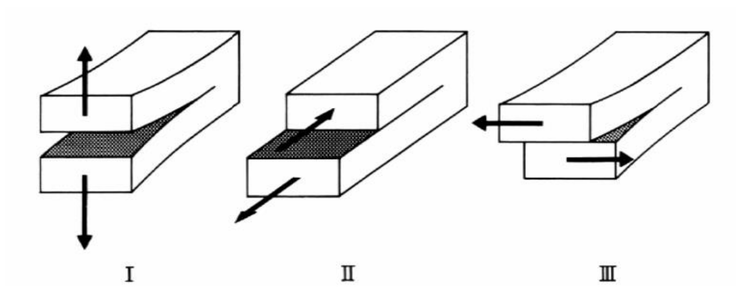


$\frac{d}{b} = 0.1 \quad \frac{d}{h} = 1 \quad K_t = 2.04 \quad M_{Max} = \frac{Ah\sigma_{Allow}}{6K_t} = 64.34N.m$

Without a hole $\quad M_{Max} = \frac{\sigma_{Allow} bh^2}{6} = 145.8N.m$

Fracture Toughness

- Fracture toughness is an indication of the amount of stress required to propagate a pre-existing flaw.
- It is a very important material property since the occurrence of flaws is not completely avoidable in the processing, fabrication, or service of a material/component.



Three crack propagation modes

Stress intensity factor

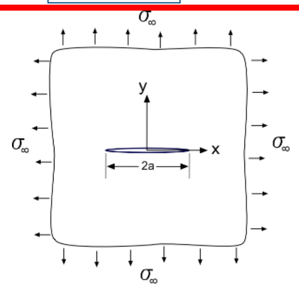
- A parameter called the stress-intensity factor (K_I) is used to determine the fracture toughness of most materials.
- K_I : A Roman numeral subscript (I) indicates the mode of fracture.

➔ Stress intensity factor for mode I: K_I

$$K_I = \sigma\sqrt{\pi a}$$

units of K :
MPa$\sqrt{m}$
or ksi$\sqrt{in}$

Crack in an infinite plate under mode I loading.



Crack tip stress is very large

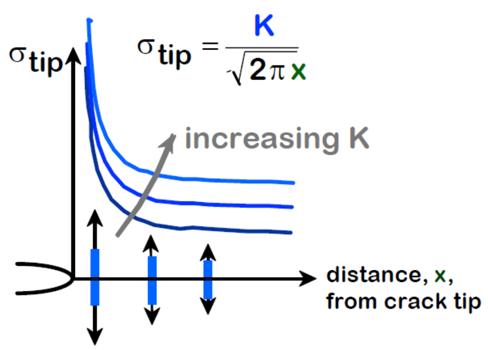
➔ Crack propagates when: the tip stress is large enough to make

Stress Intensity Factor: Depends on load & geometry

Fracture Toughness or Critical SIF: Material parameter, Depends on the material, temperature, environment, & rate of loading.

$$K_I \geq K_{Ic}$$

Crack is UNSTABLE!



Fracture toughness (Chap. 6.3)

When the magnitude of the mode I stress intensity factor reaches a critical value (i.e., K_{Ic}), crack propagation imitates!

$$K_I < K_{Ic} \Rightarrow \text{Crack is STABLE!}$$

K_{Ic}: Critical stress intensity factor = Fracture toughness

- A properly determined value of K_{Ic} represents the fracture toughness of the material independent of crack length, geometry or loading system.
- K_{Ic} is a material property

Fatigue Failure Theory

- ✓ Most failures are due to time-varying loads rather than to static loads.
- ✓ The static failure theory can fail to provide safe design guideline when loads are dynamic.
- ✓ ISSUE
  - ➔ Failure occurs at stress levels considerably lower than the yield strengths of the materials.
  - ➔ WHY?

Example: Hawaiian Boeing 737 1988 accident

Photo 2. Composite photo left side.

"Multiple site damage was the main cause of the fuselage damage. MSD can range from a few fatigue cracks among many rivet holes to the worst case of small, visually undetectable fatigue cracks on both sides of rivet holes. Even though the small crack is not visualized it may lead to catastrophic destruction of the system. In the case of B 737 after the accident, a passenger reported that as she boarded, she noticed a large vertical fuselage crack, but didn't mention it to anyone. And this small crack propagated by MSD and the final analysis revealed that MSD was most prevalent in the mid-bay areas"

Mechanism of Fatigue Failure

- Fatigue failure ALWAYS starts with CRACK.
- ALL Structural members contains discontinuities from the manufacturing or fabricating process.
- Notch or other stress concentration is the start point of fatigue cracks.

Notch (Chap. 4. 12, Chap. 8.10)

- Any geometric contour that disrupts the force flow through the part
- Hole, Groove, Fillet, etc.

- That is, any disruption to the smooth contours of a part
- Review: Geometric stress concentration factor K_t
- $\sigma_{max}=K_t \sigma_{nom}$
- For dynamic loading, MODIFY K_t based on the notch sensitivity q to obtain fatigue stress-concentration factor K_f.
- Notch sensitivity: Materials' sensitivity to stress concentrations
- Peterson (1974) $q = (K_f-1)/(K_t-1)$ where, K_t: theoretical static stress-concentration factor and K_f: fatigue stress-concentration factor → $K_f = 1+q(K_t-1)$
- First: Determine K_t
- Second: q for the given material
- Then, find K_f

Mechanism of Fatigue Failure (Chap. 8.16)

- All structural members contain disconitnuies from the manufacturing and fabricating process.
- Recall: Comet airplane failure
  → Crack starts near the corners of window (high stress concentration)
- 3 stages of fatigue failure
  1. Crack initiation
  2. Crack propagation
  3. Sudden fracture due to unstable crack growth

Crack initiation stage

- Short duration
- Assume the material is ductile
- Assume no metals are homogenous and isotropic (microscopic scale)
- Assume geometric stress concentration regions (notches)
- Significant time-varying stress (Fig. 6-1)
- The localized plastic yielding causes distortion and creates lip bands along the crystal boundaries of the material.
- LESS ductile → Develop cracks more rapidly → More notch sensitive
- Brittle material: NO yielding → skip crack initiation stage → Proceed directly to crack propagation stage

Crack propagation stage

- Involves most of the life of the part (long duration)
- Continue as long as the local stress is cycling from below the tensile yield to above the tensile yield at the crack tip
- Crack growth is due to tensile stress!!
- Crack propagation growth rate: 10^{-8} to 10^{-4} in per cycle
- Striations (줄무늬) due to each stress cycle

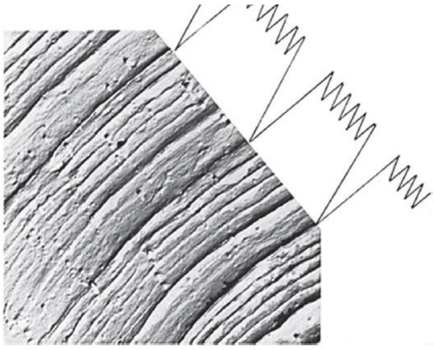


FIGURE 6-3

Fatigue Striations on the Crack Surface of an Aluminum Alloy. Spacing of the Striations Corresponds to the Cyclic Loading Pattern (From Fig. 1.5, p. 10, in D. Broek, *The Practical Use of Fracture Mechanics*, Kluwer Publishers, Dordrecht, 1988)

- Corrosion!! Another mechanism for crack propagation
- If a part containing a crack is in a corrosive environment, the crack will grow under static stress!

TWO Terms

- Stress-Corrosion or environmentally assisted crack: Combination of stress and corrosive environment
- Corrosion fatigue: the part is cyclically stressed in a corrosive environment, the crack will grow more rapidly

Fracture stage

- Instantaneous
- At some point, the crack size becomes large enough to raise the stress intensity factor at the crack tip to the level of the materials fracture toughness and sudden failure occurs instantaneously on the next tensile stress-cycle.

→ When K reaches Kc, the crack will propagate suddenly to failure.

- Beachmark (by naked eyes)
- : due to starting and stopping of the crack growth
- Beachmarks surround the origin of the crack (usually at a notch or internal stress raiser)

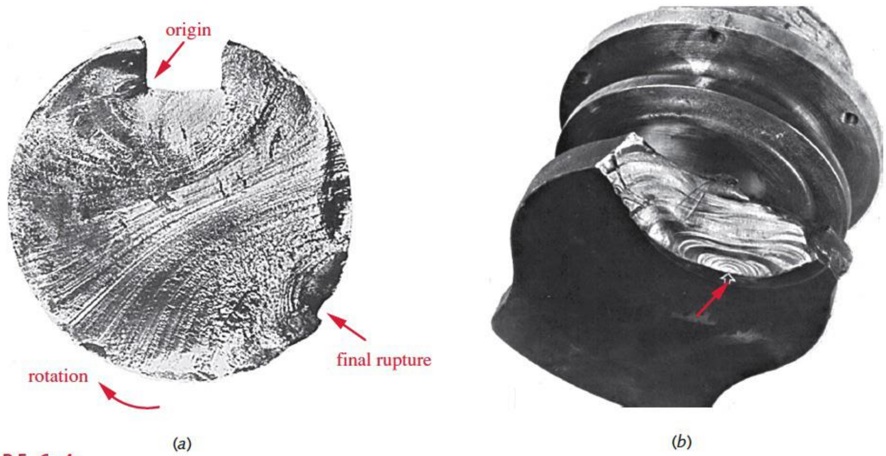


FIGURE 6-4

Two Parts That Failed in Fatigue. Note the Beachmarks: (*a*) 1040 Steel Keyed Shaft Failed in Rotating Bending. Crack Started at Keyway. (*b*) Diesel-Engine Crankshaft Failed in Combined Bending and Torsion. Crack Started at Arrow. (Source: D. J. Wulpi, *Understanding How Components Fail*. Am. Soc. for Metals: Metals Park, Ohio, 1990, Fig. 22, p. 149, and Fig. 25, p. 152.)

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