Basic Calculus

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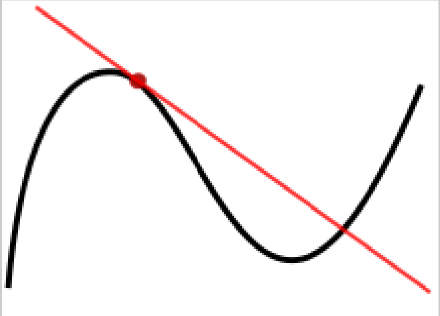
Derivative

The derivative of a function of a real variable measures the sensitivity to change of a quantity (a function or dependent variable) which is determined by another quantity (the independent variable).

The derivative measures the instantaneous rate of change of the function, as distinct from its average rate of change, and is defined as the limit of the average rate of change in the function as the length of the interval on which the average is computed tends to zero.



The graph of a function, drawn in black, and a tangent line to that function, drawn in red. The slope of the tangent line is equal to the derivative of the function at the marked point.

Differentiation

Differentiation is the action of computing a derivative.
The derivative of a function $f(x)$ of a variable x is a measure of the rate at which the value of the function changes with respect to the change of the variable. It is called the derivative of f with respect to x.
The slope m is given by

$$m = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x},$$

where the symbol Δ (Delta) is an abbreviation for "change in."

Notation

An infinitesimal change in *x* is denoted by *dx*.
The derivative of y with respect to x is written

$$\frac{dy}{dx}$$

suggesting the ratio of two infinitesimal quantities.

The above expression is read as "the derivative of y with respect to x", "d y by d x", or "d y over d x". The oral form "$d\ y\ d\ x$" is often used conversationally, although it may lead to confusion.
The derivative with respect to x of a function $f(x)$ is denoted $f'(x)$ (read as "f prime of x").

Differential (infinitesimal)

The term differential is used in calculus to refer to an infinitesimal (infinitely small) change in some varying quantity.

If y is a function of x, then the differential dy of y is related to dx by the formula

$$\mathrm{d}y = \frac{\mathrm{d}y}{\mathrm{d}x}\,\mathrm{d}x,$$

where dy/dx denotes the derivative of y with respect to x. This formula summarizes the intuitive idea that the derivative of y with respect to x is the limit of the ratio of differences $\Delta y/\Delta x$ as Δx becomes infinitesimal.

Differential of a function

The differential represents the principal part of the change in a function $y = f(x)$ with respect to changes in the independent variable. The differential dy is defined by

$$dy = f'(x)dx$$

where $f'(x)$ is the derivative of f with respect to x, and dx is an additional real variable (so that dy is a function of x and dx).

The notation is such that the equation

$$dy = \frac{dy}{dx}\,dx$$

Differentials in several variables

For functions of more than one independent variable,

$$y = f(x_1, \ldots, x_n),$$

the partial differential of y with respect to any one of the variables x_1 is the principal part of the change in y resulting from a change dx_1 in that one variable. The partial differential is therefore $\frac{\partial y}{\partial x_1} dx_1$

involving the partial derivative of y with respect to x_1. The sum of the partial differentials with respect to all of the independent variables is the total differential

$$dy = \frac{\partial y}{\partial x_1} dx_1 + \cdots + \frac{\partial y}{\partial x_n} dx_n,$$

which is the principal part of the change in y resulting from changes in the independent variables x_i.

A partial derivative such as $\frac{\partial z}{\partial x}$ (read as "the partial derivative of z with respect to x)

Second derivative

In calculus, the second derivative, or the second order derivative, of a function f is the derivative of the derivative of f.

Roughly speaking, the second derivative measures how the rate of change of a quantity is itself changing.

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{d^2\boldsymbol{x}}{dt^2}$$

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Thank you!